Chapter 5:
Diffraction and Beam Formation Using Arrays

B-mode Imaging

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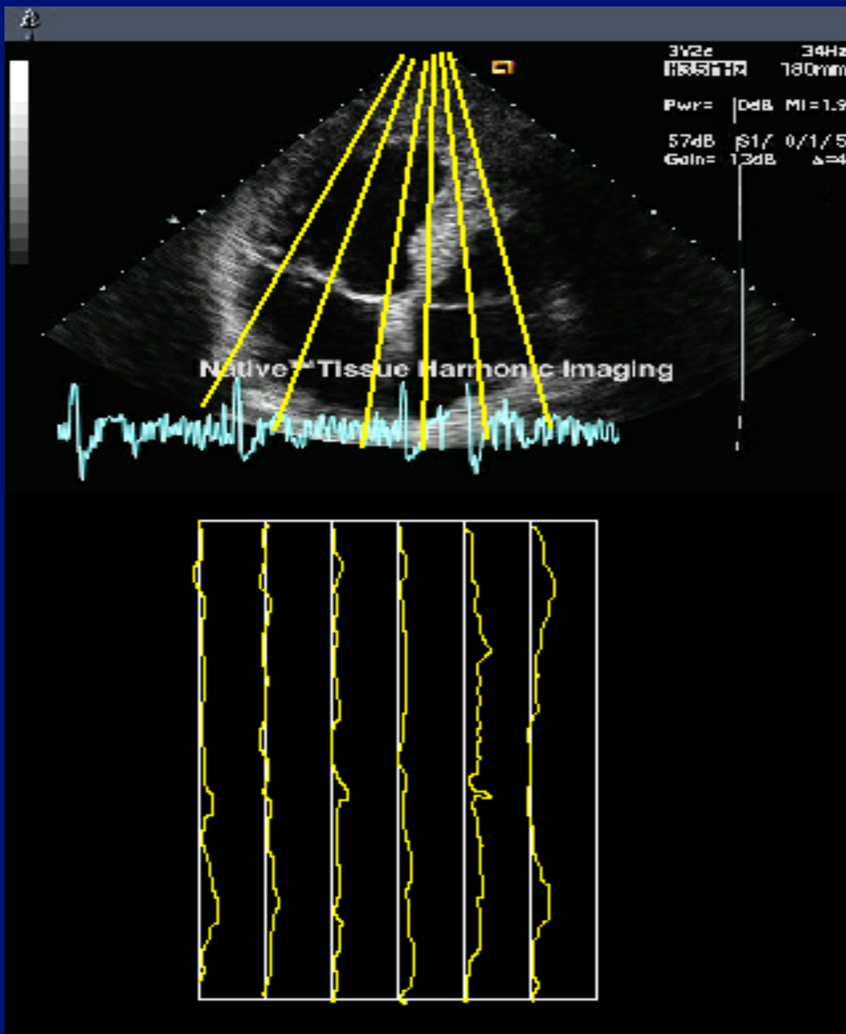
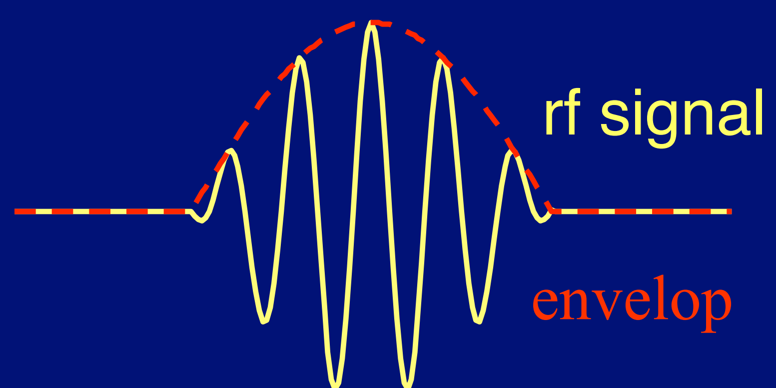
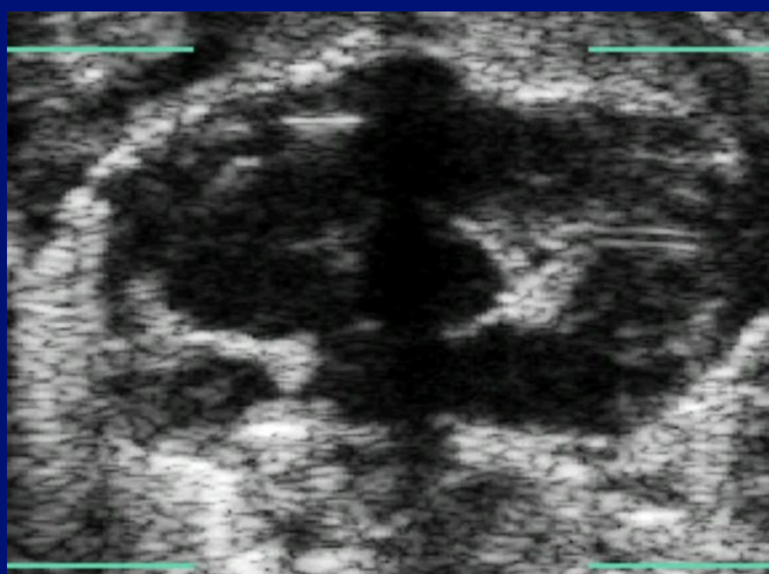
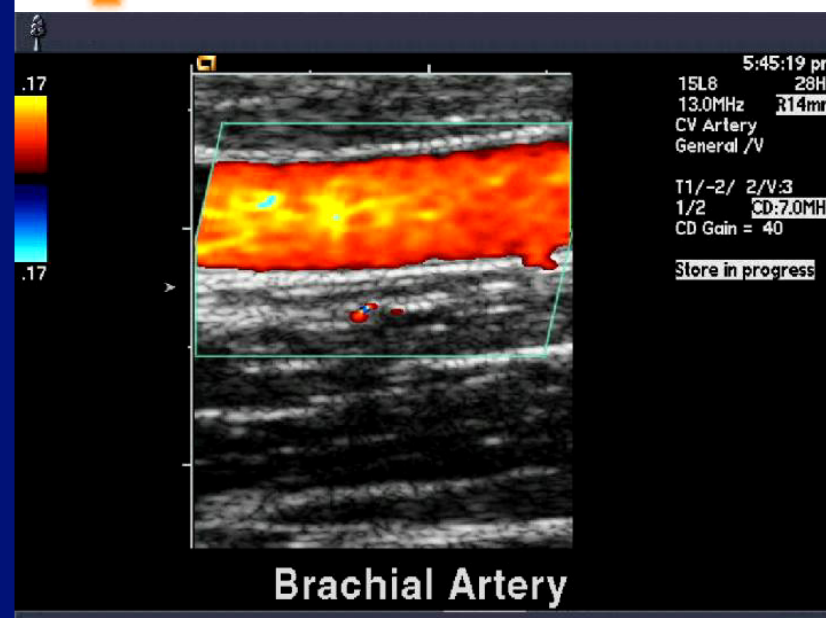




$$p\left(t-\frac{2z'}{c}\right)=A\left(t-\frac{2z'}{c}\right)\cos\left(2\pi f_0\left(t-\frac{2z'}{c}\right)\right)$$

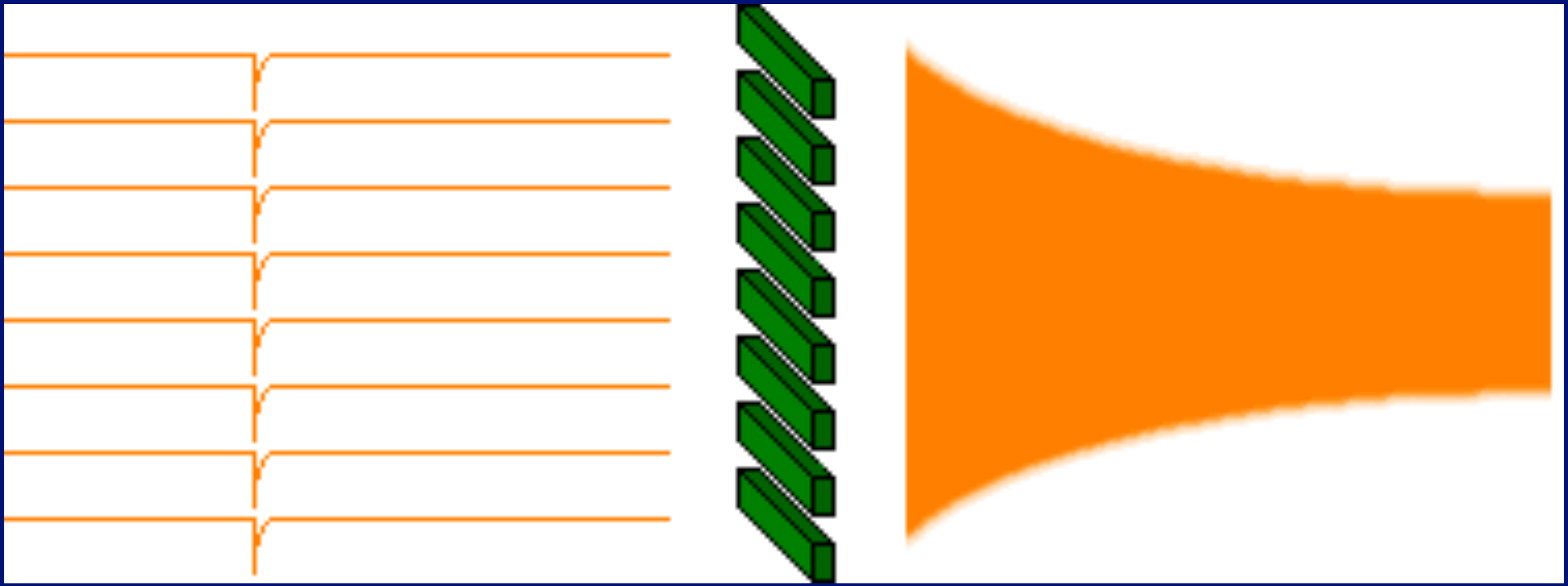
Linear Scanning



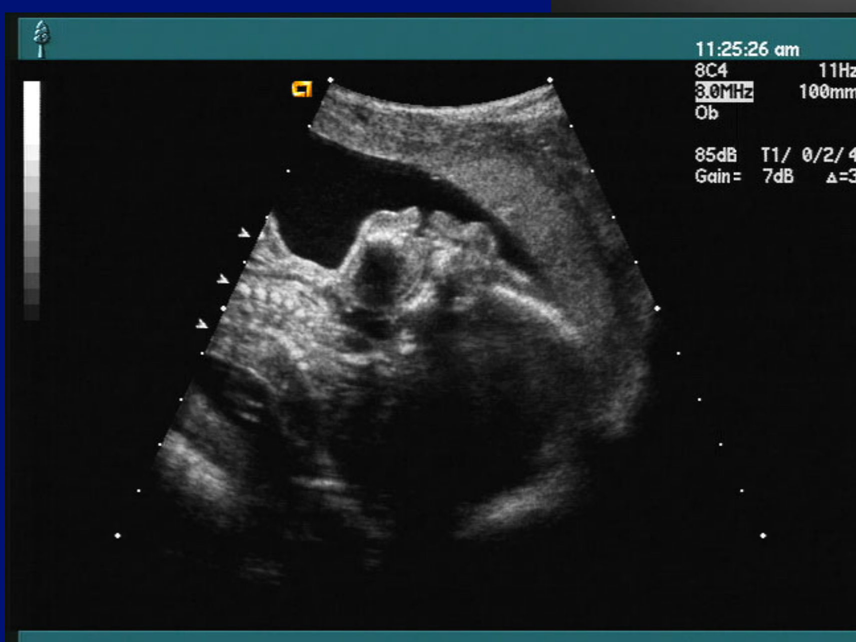


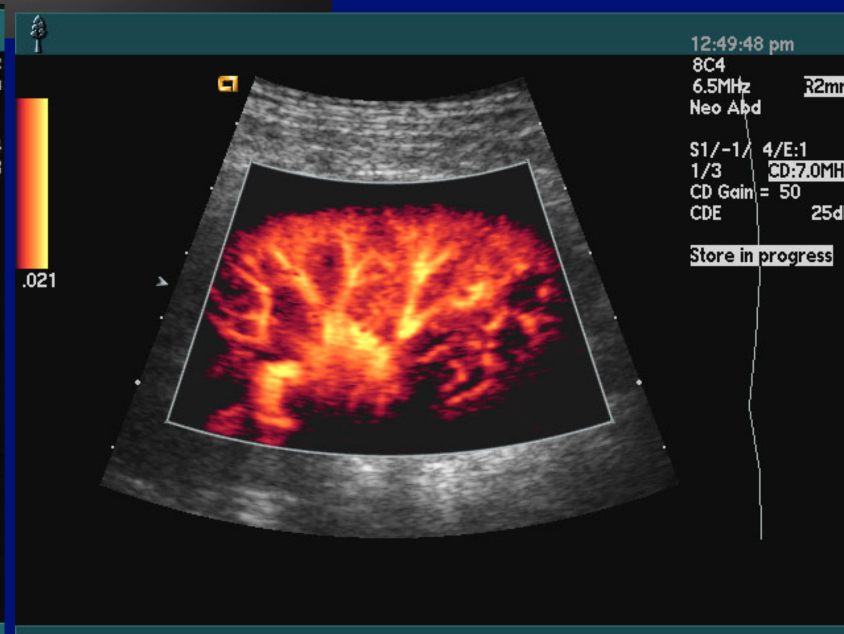
Beam Formation Using Arrays

Focusing:

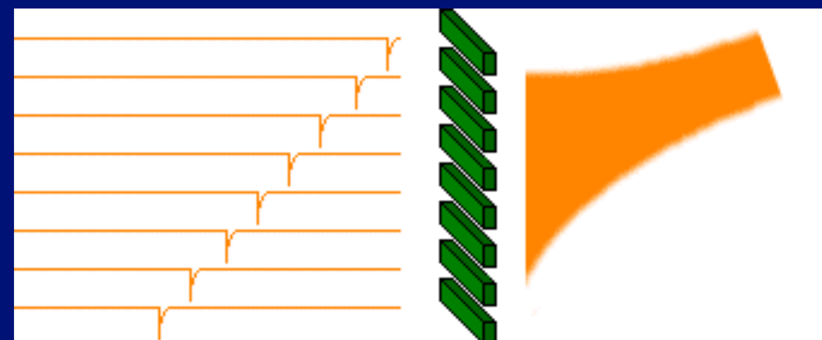


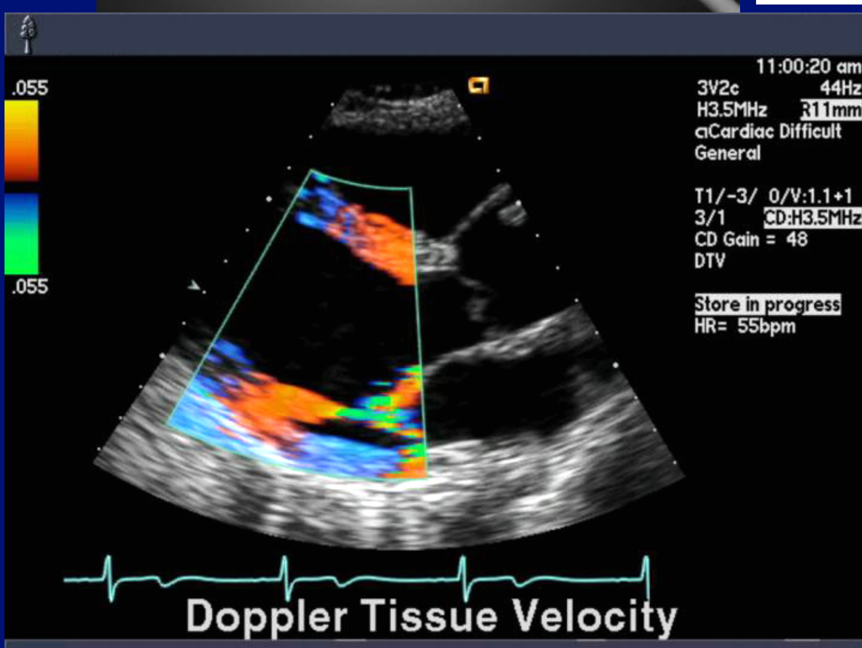
Curved Linear Scanning

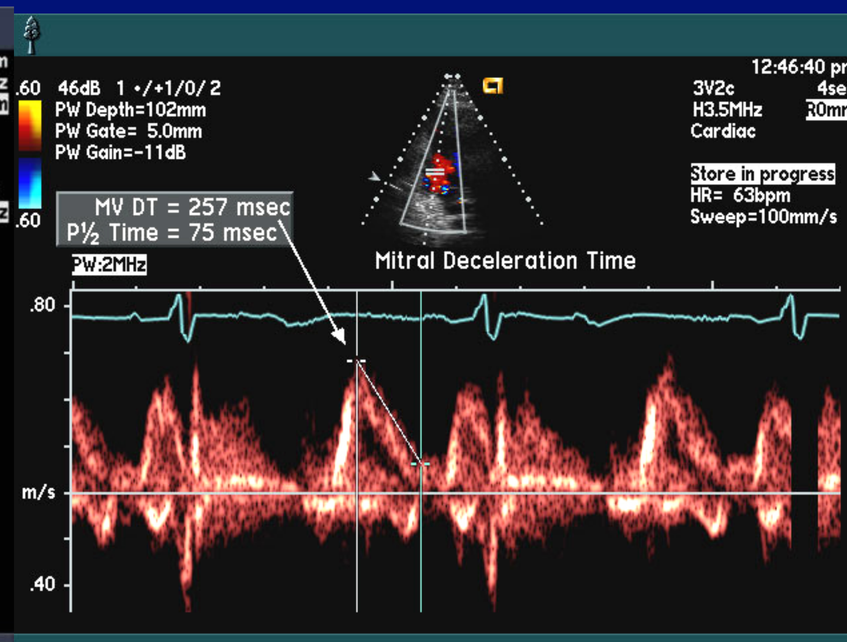




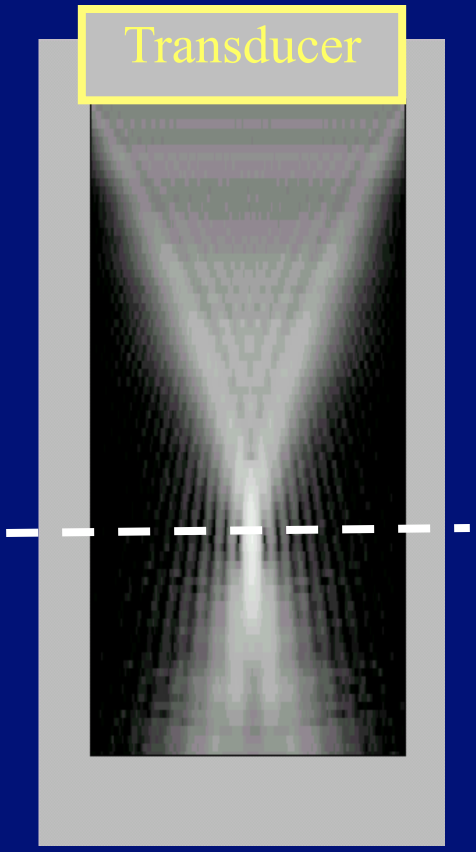
Sector Steering



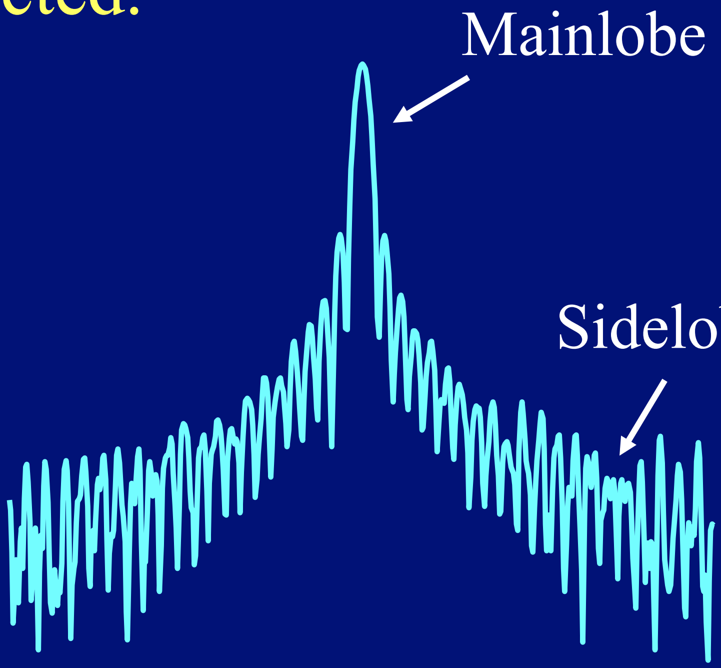




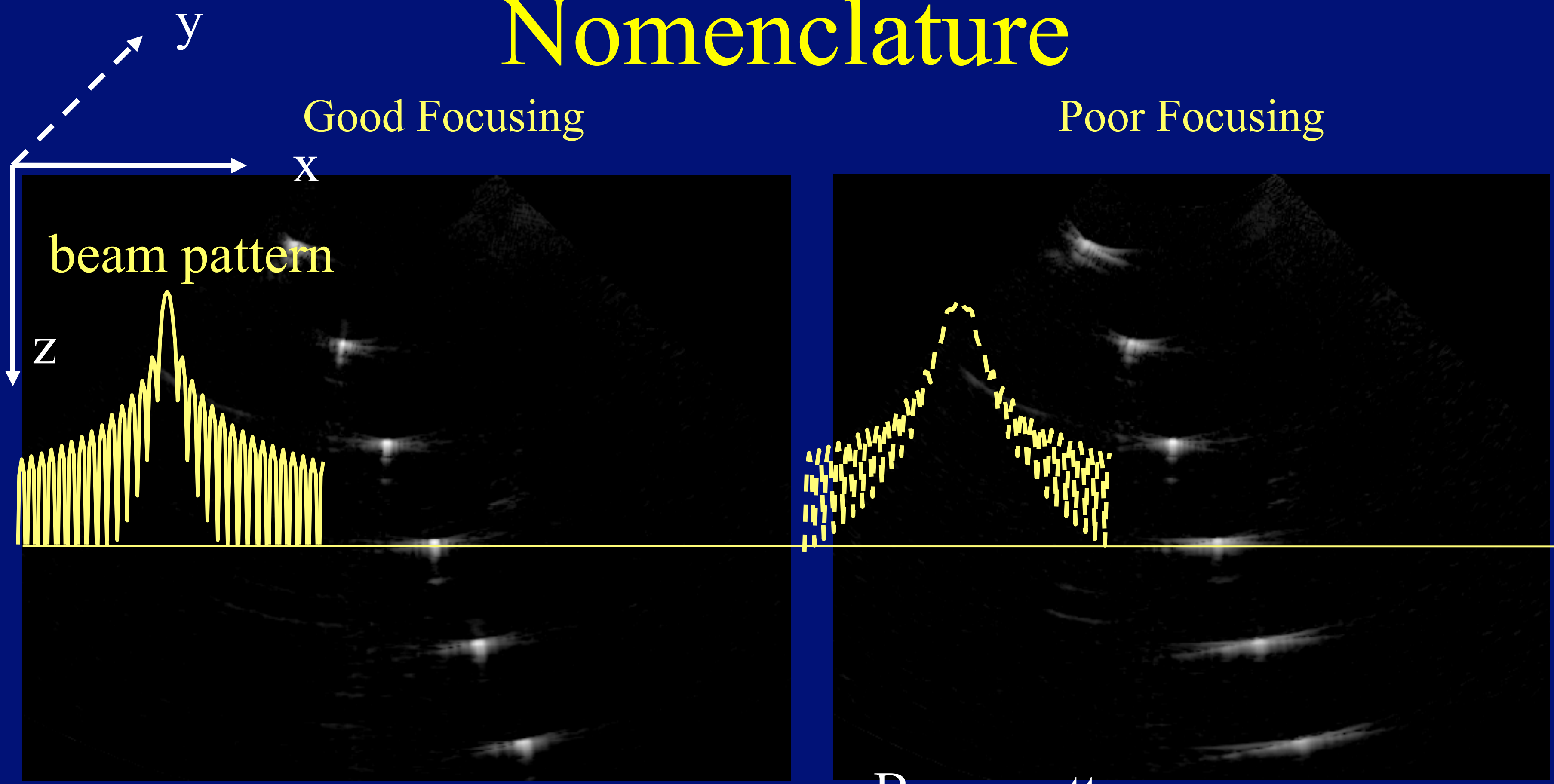
Focusing ←→ Beam Formation



- To form a beam of sound wave such that only the objects along the beam direction are illuminated and possibly detected.

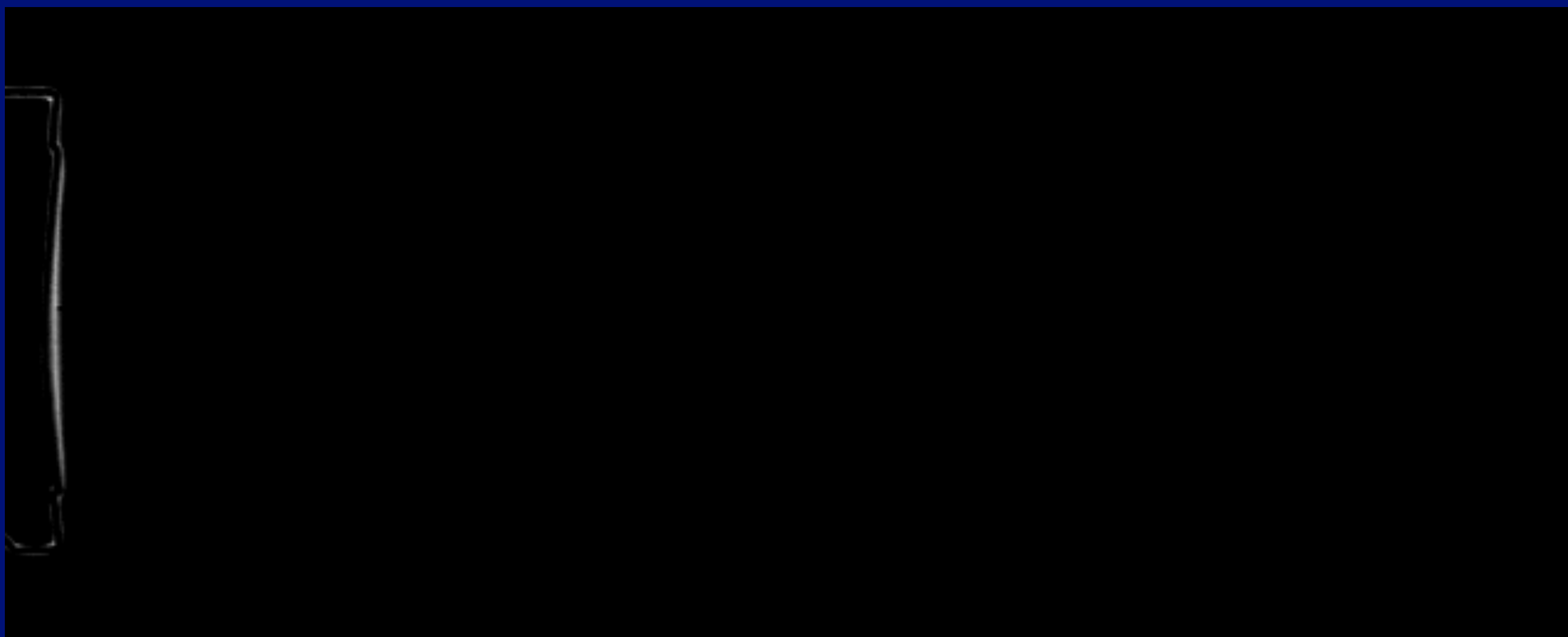


Nomenclature



x: Lateral, azimuthal, scan
y: Elevational, non-scan
z: Axial, range, depth

Beam pattern
Radiation pattern
Diffraction pattern
Focusing pattern



Beamforming

- Manipulation of transmit and receive apertures.
- Trade-off between performance/cost to achieve:
  - Steer and focus the transmit beam.
  - Dynamically steer and focus the receive beam.
  - Provide accurate delay and apodization.
  - Provide dynamic receive control.

Imaging Model

transmitter → transducer → propagation in the body → transducer → receiver → display

A-scan:

$$V(t) = k\iiint \frac{R(x',y',z')e^{-2\beta z'}}{z'} B(x',y',z')\,p\left(t-\frac{2z'}{c}\right)dx'dy'dz'$$

B-scan:

$$S(x,t) = k\iiint R(x',y',z')B(x'-x,y',z')\,p\left(t-\frac{2z'}{c}\right)dx'dy'dz'$$

Scanning → Convolution

(Correlation vs. Convolution)

Imaging Model

$$p(t-\frac{2z'}{c}) = A(t-\frac{2z'}{c})\cos(2\pi f_0(t-\frac{2z'}{c}))$$

Ideally,

$$S(x,t) = R(x, y_0, ct/2)$$

In practice,

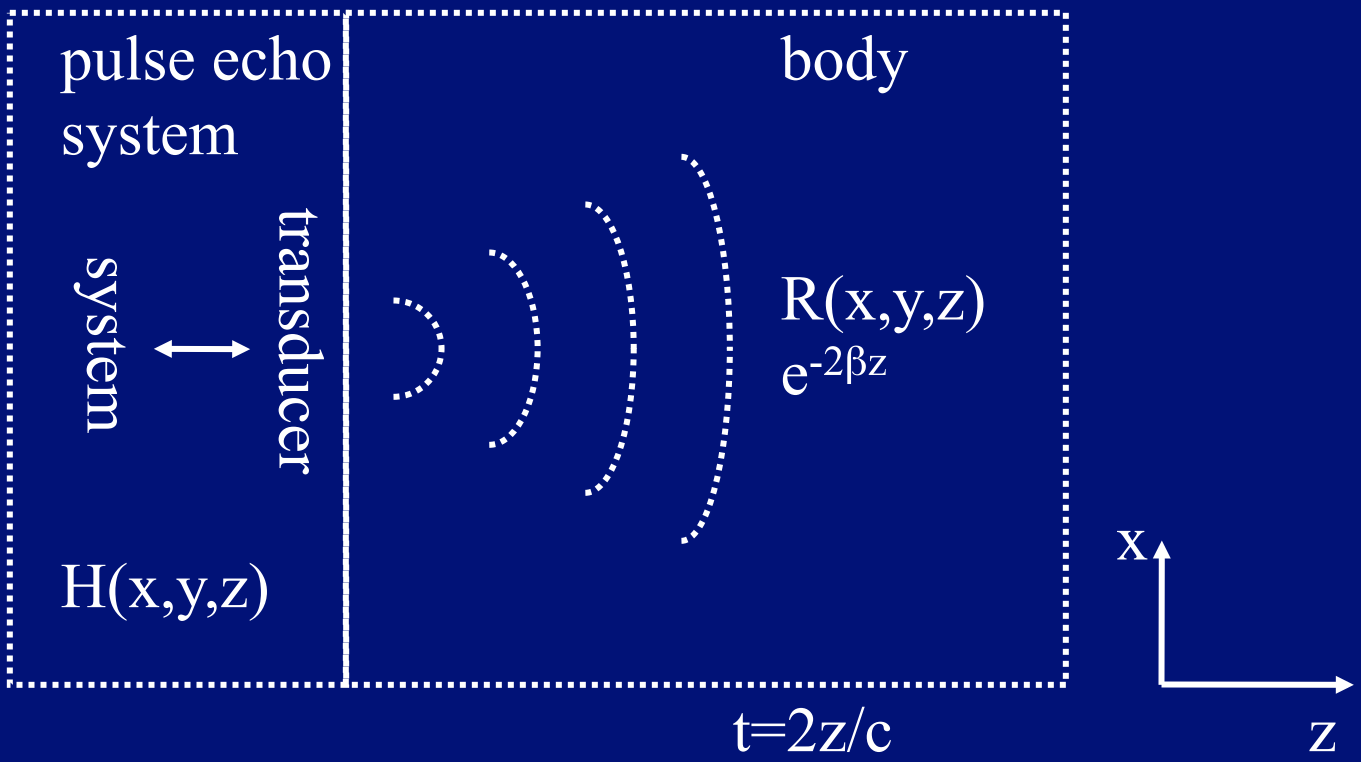
$B(\times)$: determined by diffraction

$A(\times)$: determined by transducer bandwidth

Beam Formation as Spatial Filtering

- Propagation can be viewed as a process of linear filtering (convolution).
- Beam formation can be viewed as an inverse filter (or others, such as a matched filter).

Imaging Model

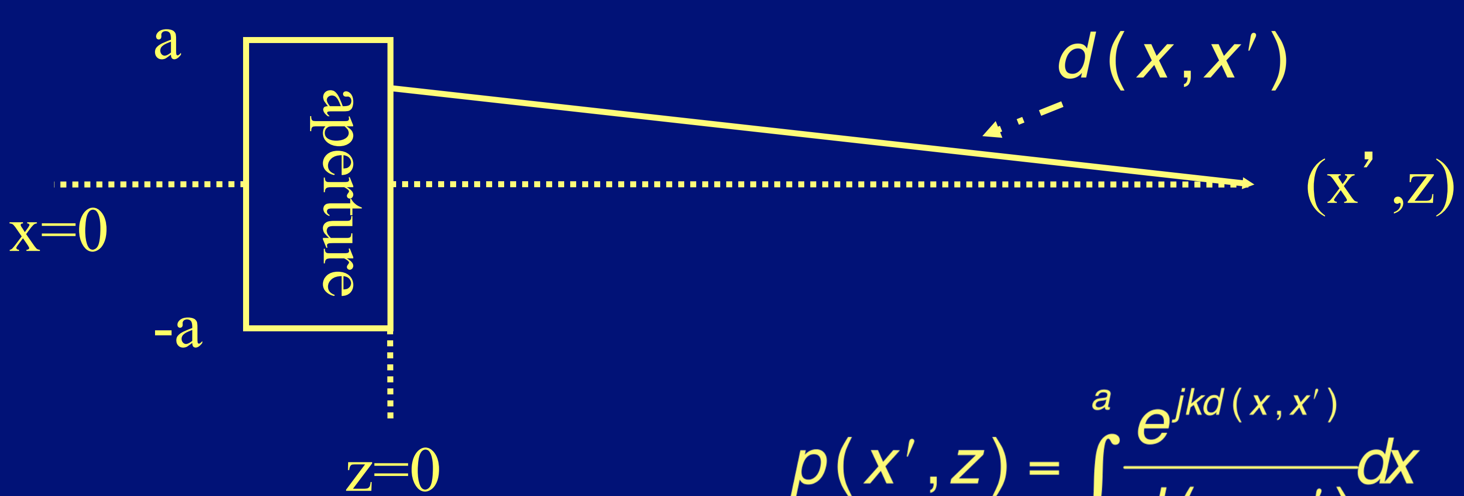


$$S(x,y,z) = H(x,y,z) *** R(x,y,z)\Big|_{z=ct/2}$$

Diffraction from 1D Apertures

- Free space Green's function:

$$p(R) = A_0 \frac{e^{jkR}}{R}$$



$$p(x', z) = \int_{-a}^{a} \frac{e^{jkd(x, x')}}{d(x, x')} dx$$

Continuous wave
(CW, single frequency)

In the Fresnel Region

$$z^2 >> (x - x')^2$$

$$d(x, x') = z\left(1 + \frac{(x - x')^2}{z^2}\right)^{1/2} \approx z + \frac{(x - x')^2}{2z}$$

$$p(x', z) \approx \frac{1}{z}\int_{-a}^{a} e^{jkz} e^{jk(x-x')^2/2z} dx = \frac{e^{jkz} e^{jkx'^2/2z}}{z}\int_{-a}^{a} e^{-jkxx'/z} e^{jkx^2/2z} dx$$

$$C(x) = |C(x)| e^{j\theta(x)}$$

$$p(x', z) \approx \frac{e^{jkz} e^{jkx'^2/2z}}{z}\int_{-a}^{a} C(x) e^{-jkxx'/z} e^{jkx^2/2z} dx$$

Focusing in the Far Field (or Focal Point)

$ka^2 / 2z << 1$

$$p(x',z) \approx \frac{e^{jkz} e^{jkx'^2/2z}}{z} \int_{-a}^{a} C(x) e^{-jkxx'/z} dx = \frac{e^{jkz} e^{jkx'^2/2z}}{z} F.T.[C(x)]$$

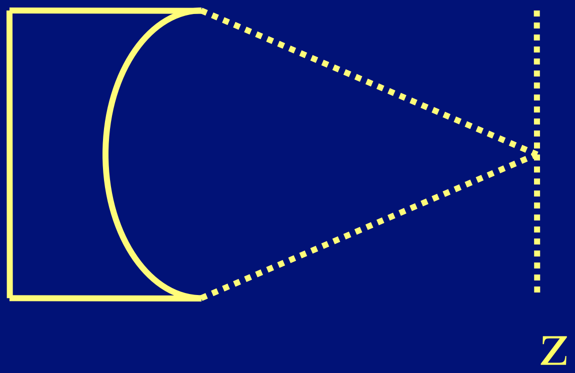
Aperture ←(F.T.)→ Radiation Pattern

When not in the far field → effective aperture function

$$C(x) = |C(x)| e^{-jkx^2/2z}$$

Focusing: An Acoustic Lens

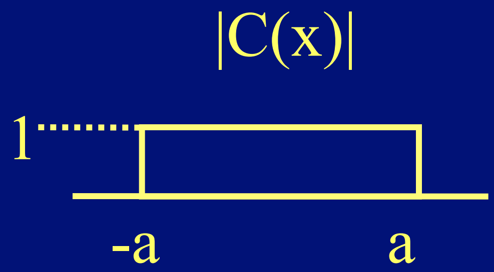
$$C(x) = |C(x)| e^{-jkx^2/2z}$$

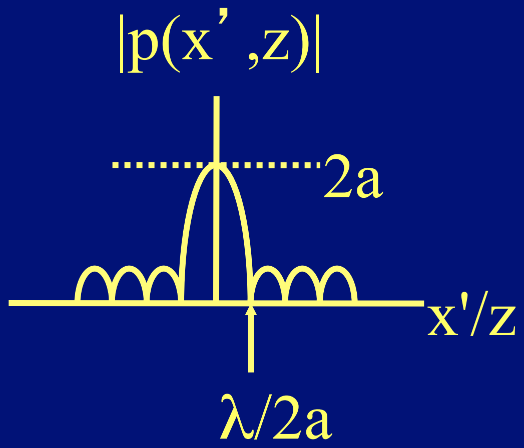


When out of the fixed focal point:

$$C'(x) = |C(x)| e^{\frac{jkx^2}{2}\left(\frac{1}{z}-\frac{1}{z_0}\right)}$$

Radiation Pattern of a Rectangular Aperture



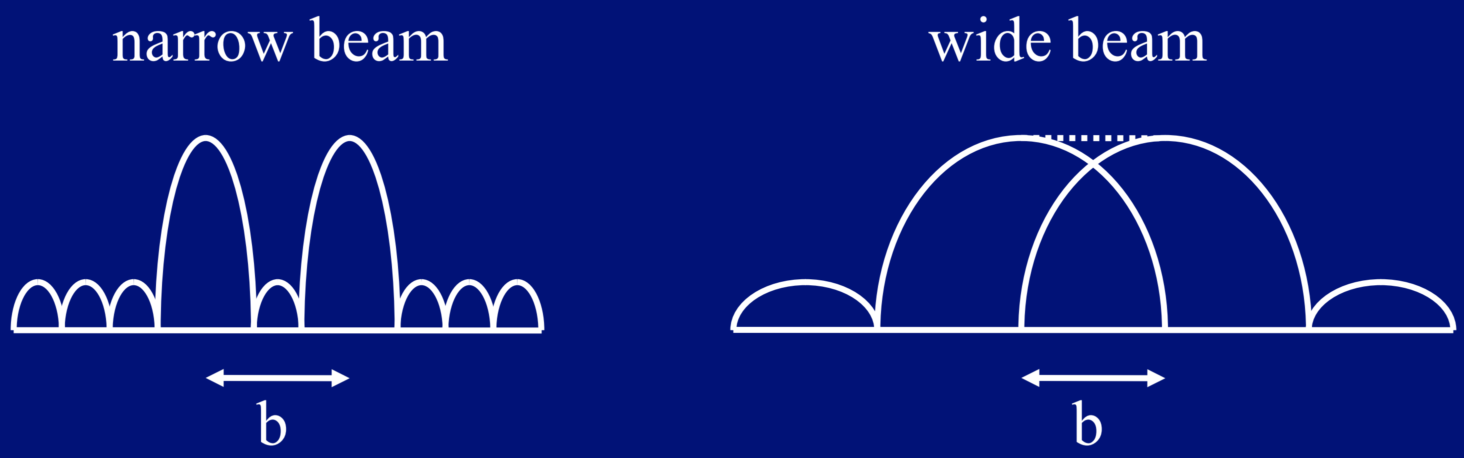


Beam width vs. Aperture size and frequency

$$\left|p(x',z)\right| = \left|\int_{-a}^{a} e^{-jkxx'/z}\,dx\right| = \left|\frac{1}{jkx'/z}\left[e^{jkx'a/z} - e^{-jkx'a/z}\right]\right| = \left|2a\frac{\sin kx'a/z}{kx'a/z}\right| = \left|2a\sin c(\frac{kx'a}{z})\right|$$

Lateral Resolution

- Frequency ↑
- Aperture size ↑
- -3 dB, -6 dB, -10 dB, -20 dB,…etc.

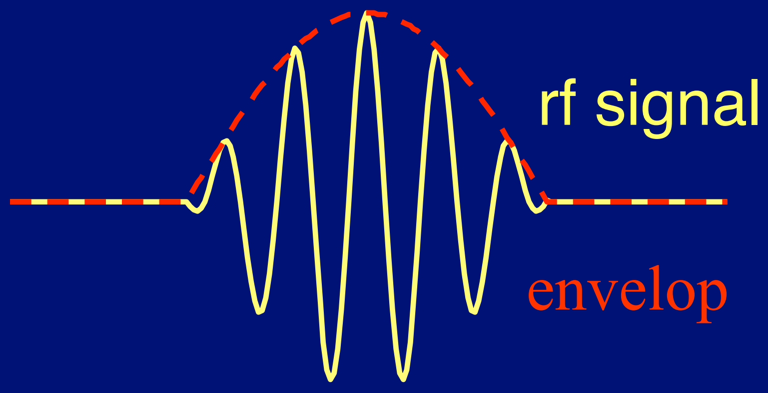


CW to Wideband

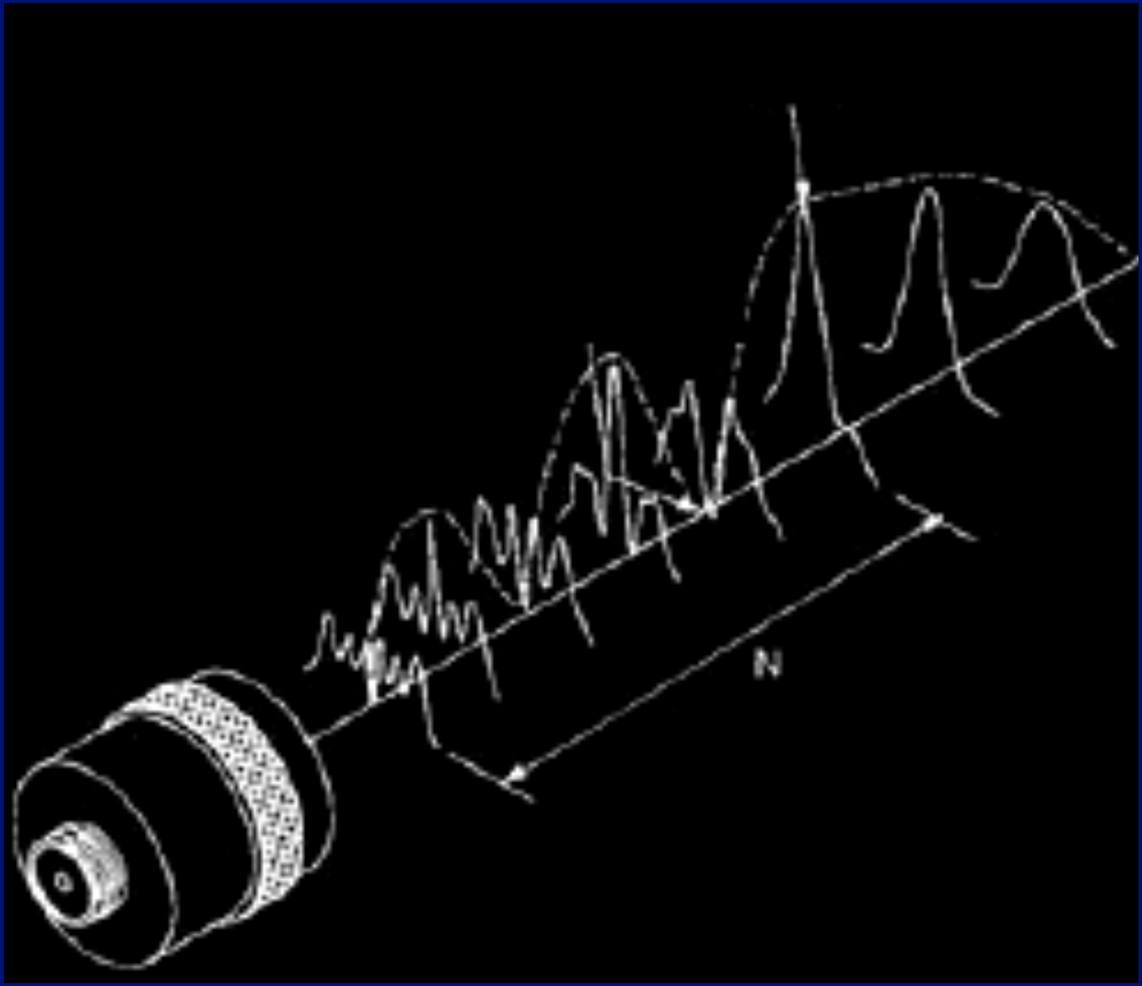
$$B(x',z)=\int T(x',z,\omega)\,R(x',z,\omega)\,A(\omega)\,d\omega$$

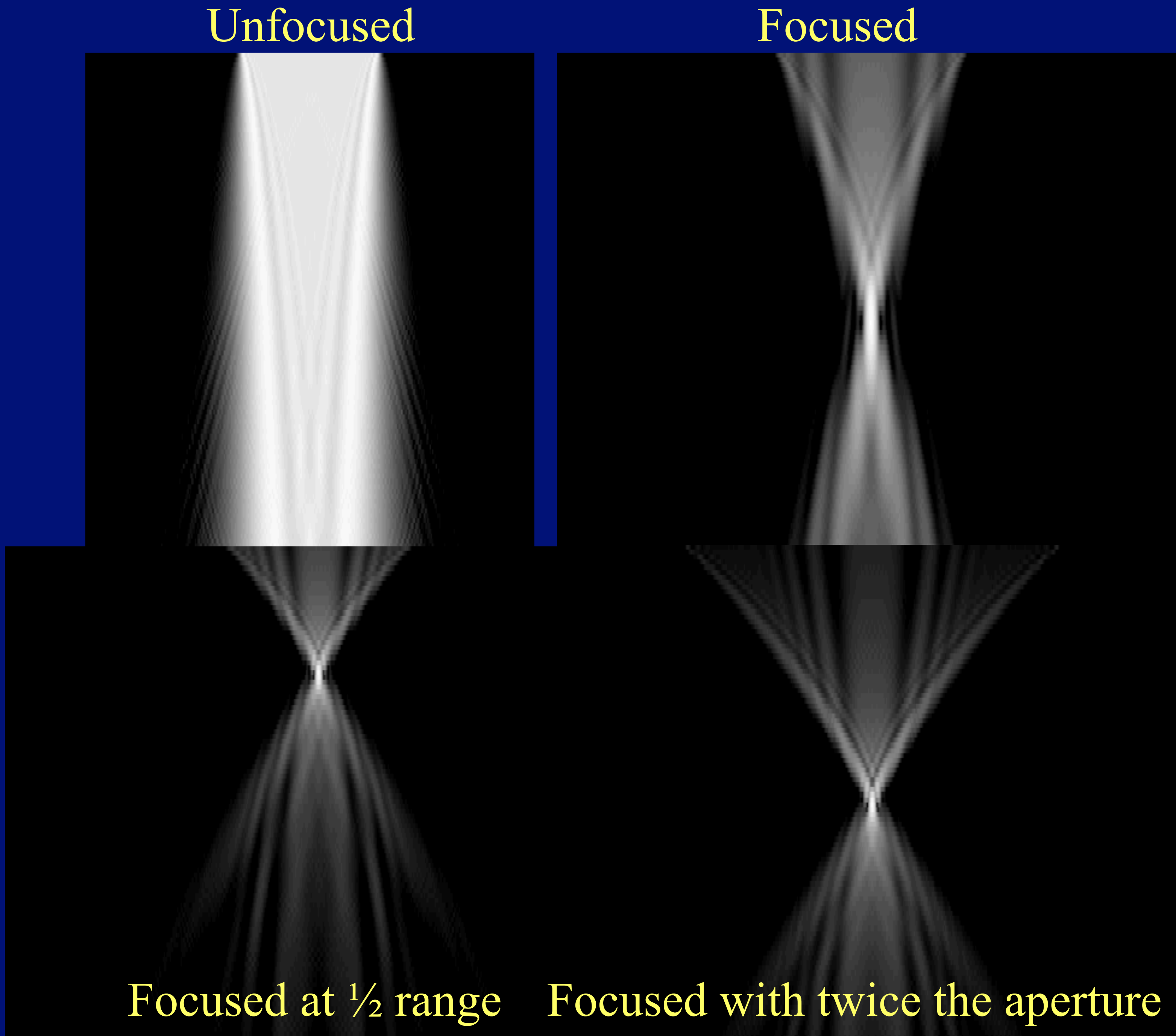
$$A(t)\Leftrightarrow A(\omega)$$

$$p(t-\frac{2z'}{c})=A(t-\frac{2z'}{c})\cos(2\pi f_0(t-\frac{2z'}{c}))$$

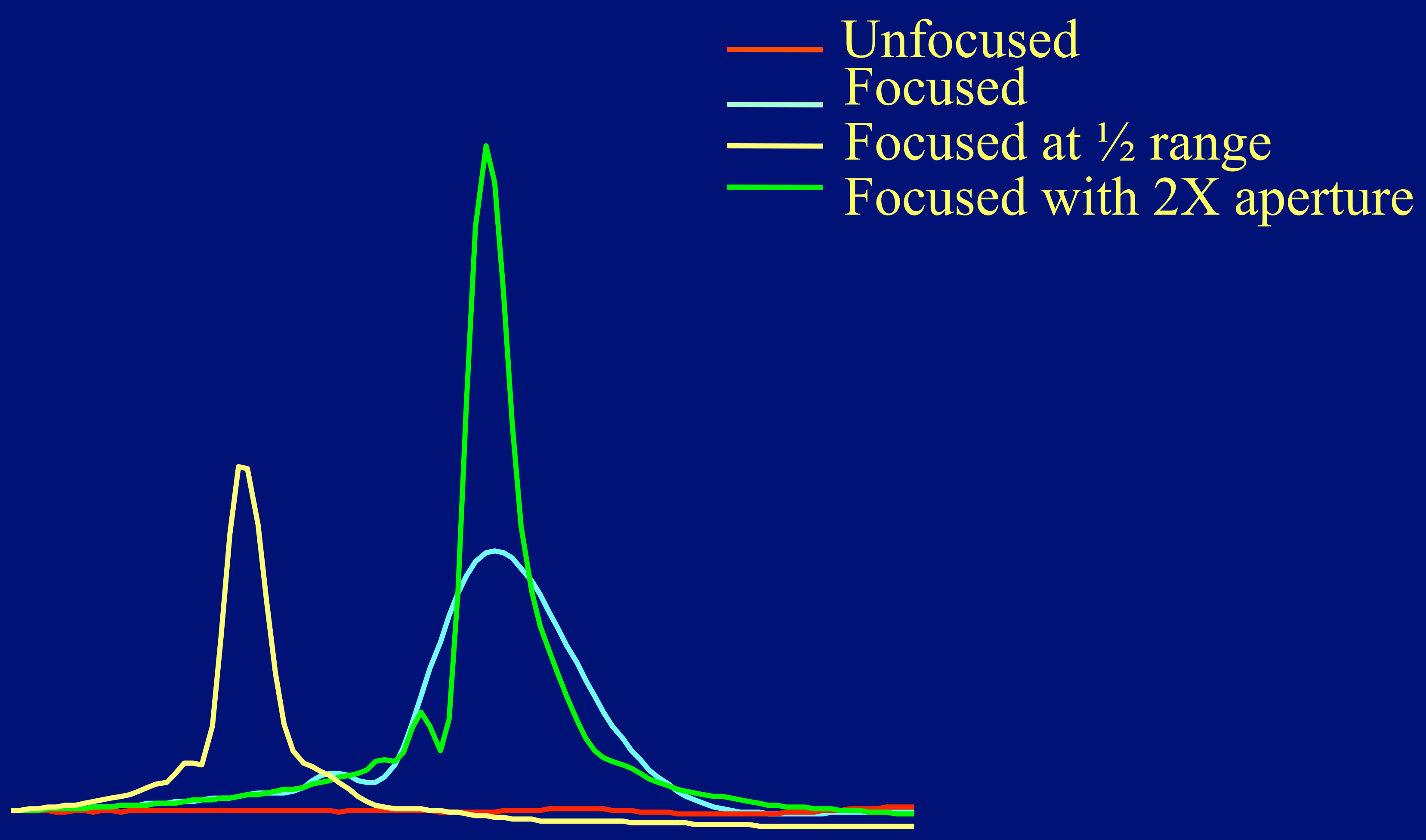


Radiation Pattern

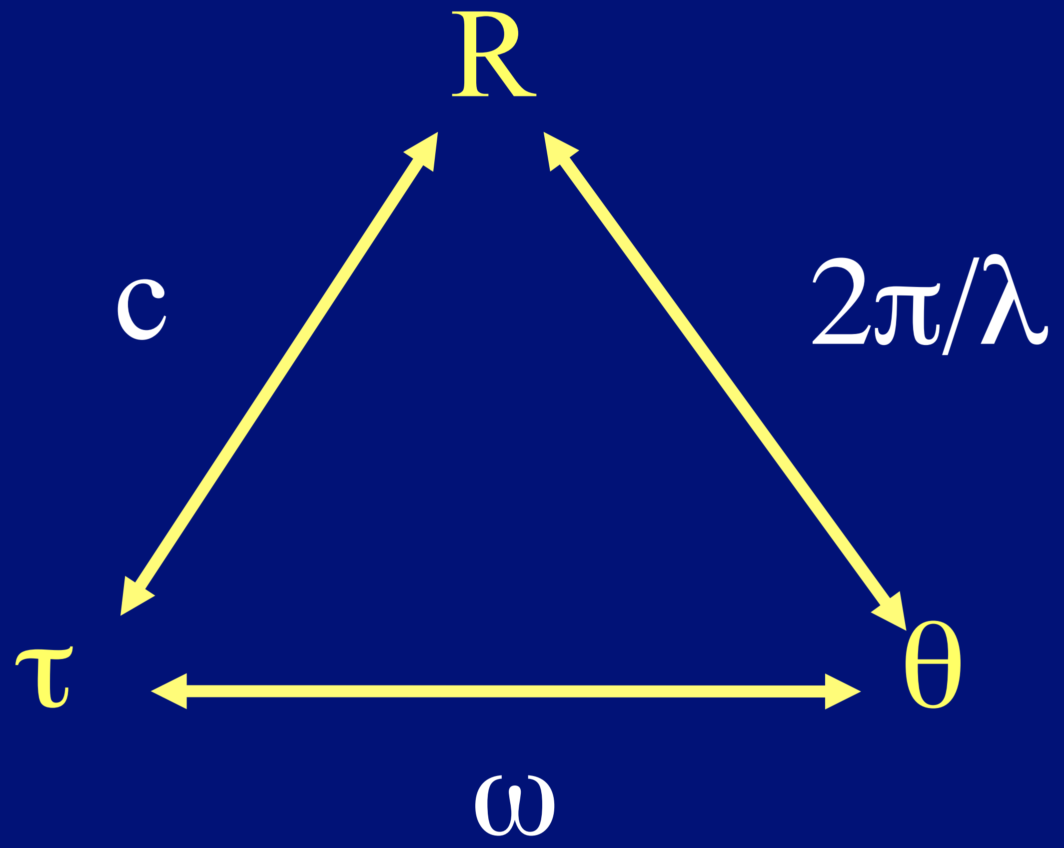



Unfocused
Focused
Focused at ½ range
Focused with twice the aperture

Axial Intensity



Diffraction and Propagation Delays


R
c
2π/λ
τ
θ
ω

CW to Pulse Wave

$$p(x',z,t) = \int_{-a}^{a} A(t-\tau(x,x',z))\cos\omega_0(t-\tau(x,x',z))\,dx$$

$$\tau(x,x',z) = \left((x-x')^2 + z^2\right)^{\frac{1}{2}}/c$$

In Fresnel region $\tau(x,x',z) \approx \dfrac{z}{c} + \dfrac{(x-x')^2}{2zc}$

Apply focusing delays $\tau'(x,x',z) = \dfrac{x^2}{2zc}$

$$p(x',z,t) = \int_{-a}^{a} A(t-\tau(x,x',z)+\tau'(x,x',z))\cos\omega_0(t-\tau(x,x',z)+\tau'(x,x',z))\,dx$$

$$= \int_{-a}^{a} A\left(t-\frac{z}{c}+\frac{xx'}{zc}-\frac{x'^2}{2zc}\right)\cos\omega_0\left(t-\frac{z}{c}+\frac{xx'}{zc}-\frac{x'^2}{2zc}\right)dx$$

CW to Pulse Wave

$x' = 0$

$$p(0,z,t) = \int_{-a}^{a} A(t-\frac{z}{c})\cos\omega_0(t-\frac{z}{c})\,dx = 2aA(t-\frac{z}{c})\cos\omega_0(t-\frac{z}{c})$$

$x' \neq 0$

$$p(x',z,t) = \mathrm{Re}\left\{\int_{-a}^{a} A(t-\frac{z}{c}+\frac{xx'}{zc}-\frac{x'^2}{2zc})\,e^{j\omega_0(t-\frac{z}{c}+\frac{xx'}{zc}-\frac{x'^2}{2zc})}dx\right\}$$

$$= \mathrm{Re}\left\{e^{j\omega_0(t-\frac{z}{c}-\frac{x'^2}{2zc})}\int_{-a}^{a} A(t-\frac{z}{c}+\frac{xx'}{zc}-\frac{x'^2}{2zc})\,e^{j\omega_0\frac{xx'}{zc}}dx\right\}$$

$$\frac{xx'}{zc}-\frac{x'^2}{2zc}\approx 0$$

$$p(x',z,t) = \mathrm{Re}\left\{e^{j\omega_0(t-\frac{z}{c}-\frac{x'^2}{2zc})}A(t-\frac{z}{c})\int_{-a}^{a} e^{j\omega_0\frac{xx'}{zc}}dx\right\}$$

Beam Formation Using Arrays

$$O(t) = \sum_{i=1}^{N} S_i\,(t - \tau\,(x_i\,, R, \theta\,))$$

$$\frac{x'}{z} \Rightarrow \sin\theta$$

Propagating Delays

$$\tau(x_i, R, \theta) = \frac{\left((x_i - R\sin\theta)^2 + R^2\cos^2\theta\right)^{1/2}}{c} = \frac{R}{c}\left(1 + \frac{x_i^2}{R^2} - \frac{2x_i}{R}\sin\theta\right)^{1/2}$$

In Fresnel region

$$\tau(x_i, R, \theta) \approx \frac{R}{c}\left(1 + \frac{x_i^2}{2R^2} - \frac{x_i}{R}\sin\theta - \frac{x_i^2}{2R^2}\sin^2\theta\right)$$

$$= \frac{R}{c}\left(1 - \frac{x_i}{R}\sin\theta + \frac{x_i^2}{2R^2}\cos^2\theta\right) = \frac{R}{c} - \frac{x_i\sin\theta}{c} + \frac{x_i^2\cos^2\theta}{2Rc}$$

Effective aperture size: $2a \longrightarrow 2a\cos\theta$

Propagating Delays

Transmit:

$$\tau^{T}(x_i, R, \theta) = -\frac{x_i \sin\theta}{c} + \frac{x_i^2 \cos^2\theta}{2Rc}$$

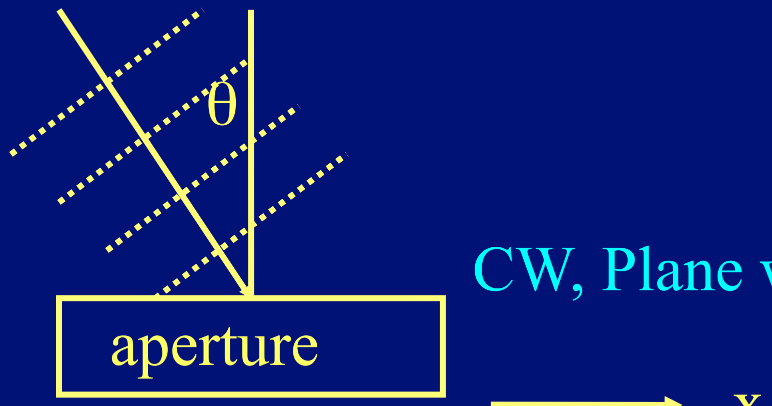
Receive:

$$\tau^{R}(x_i, R, \theta) = \frac{2R}{c} - \frac{x_i \sin\theta}{c} + \frac{x_i^2 \cos^2\theta}{2Rc}$$

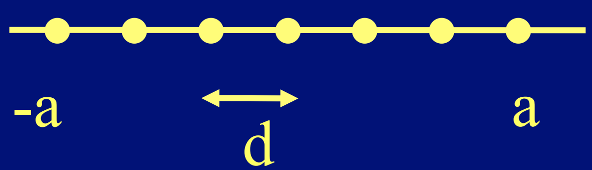
Beam Forming Using Arrays

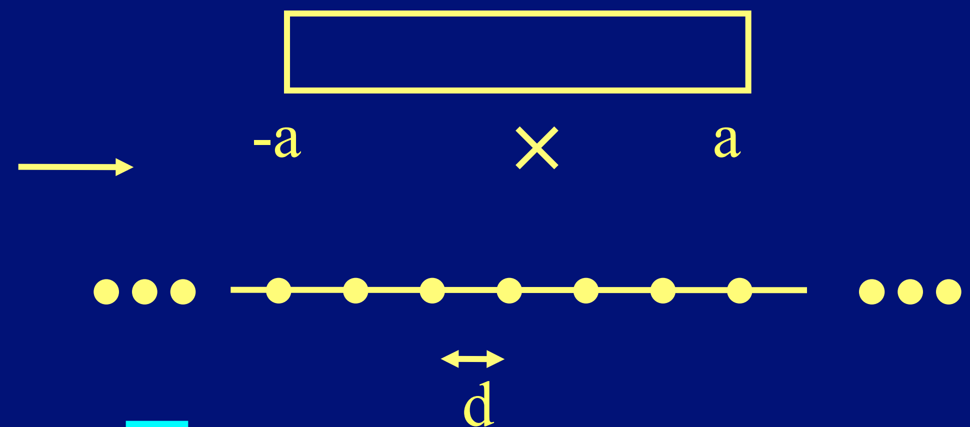
$$\frac{x'}{z} \Rightarrow \sin\theta$$

$$p(\theta) = \int_{-a}^{a} C(x)\, e^{-jkx\sin\theta}\, dx$$



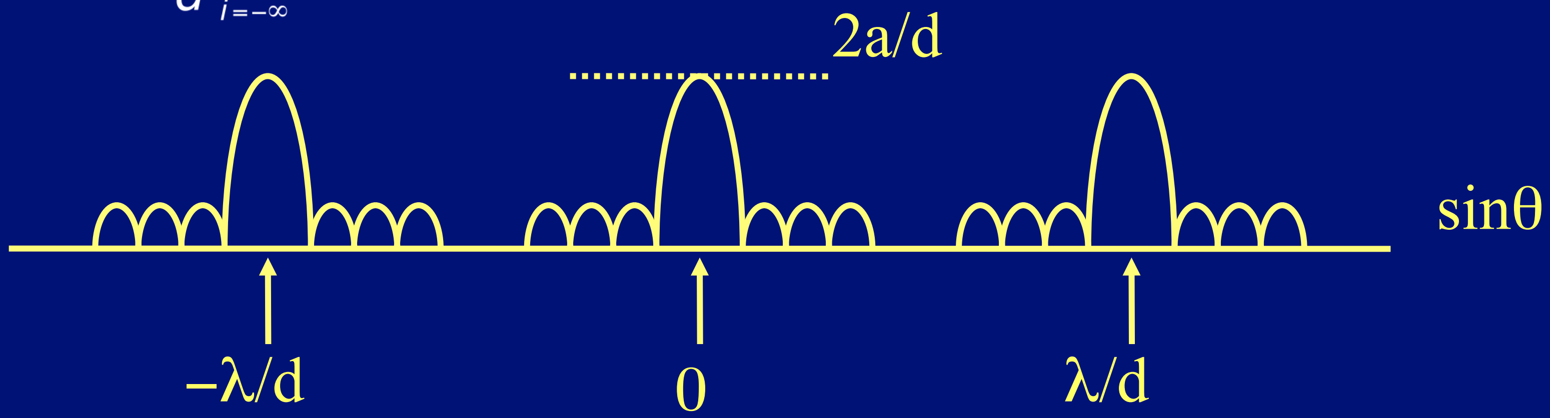
Radiation Pattern of a Sampled Aperture (I)



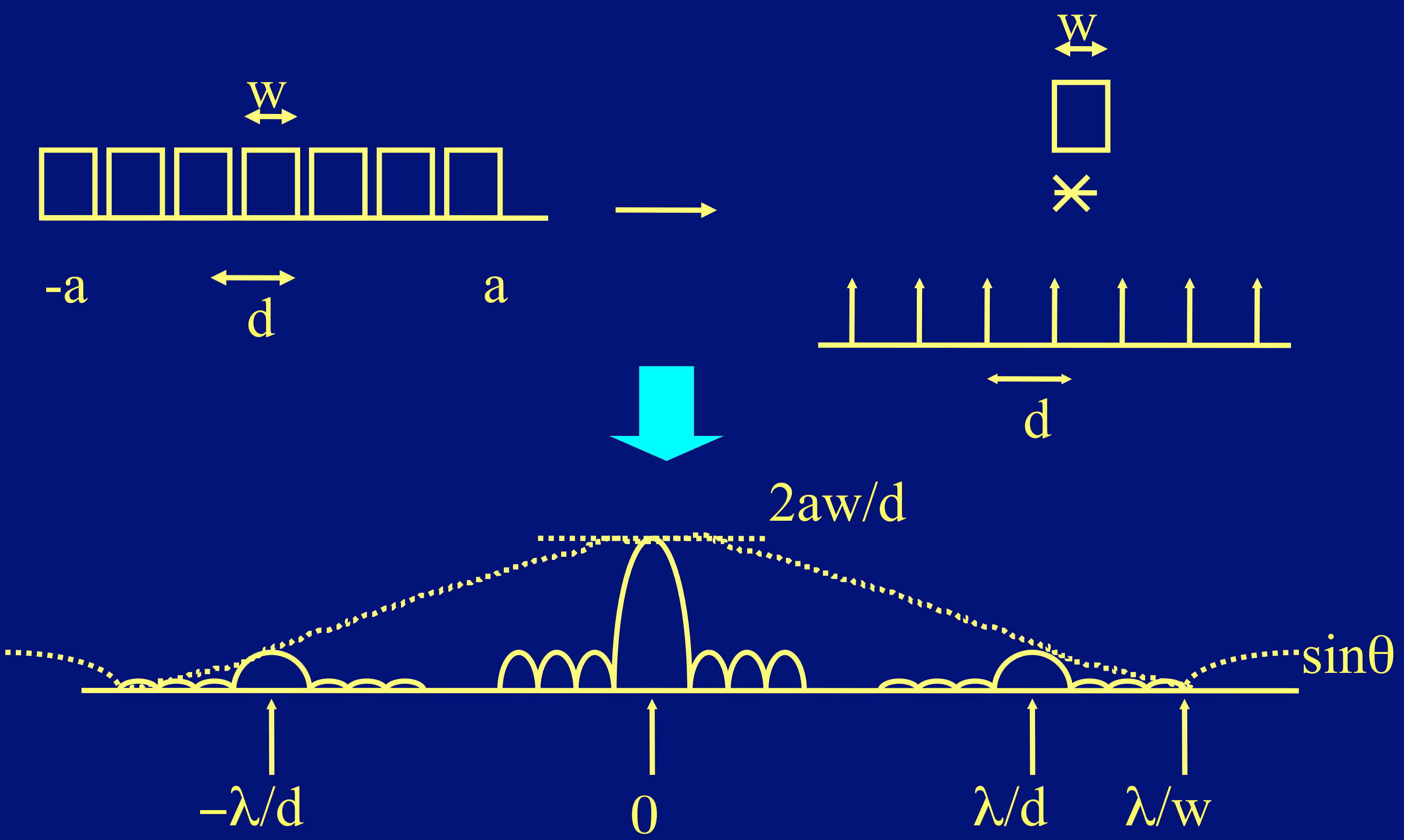


$$f = \sin\theta / \lambda$$

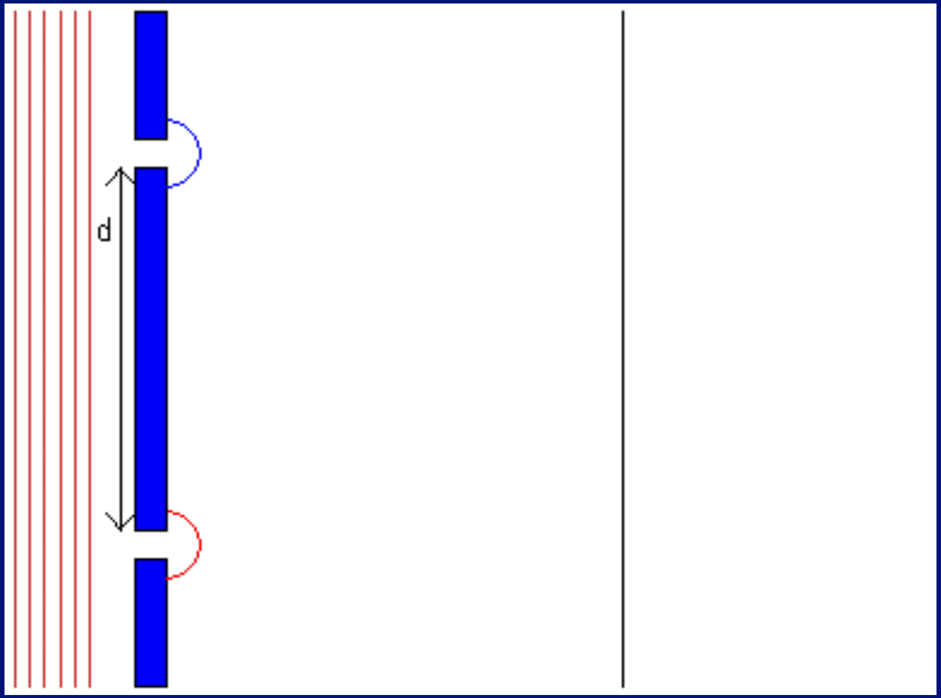
$$\sum_{i=-\infty}^{\infty} \delta(x - id) \leftrightarrow \frac{1}{d} \sum_{i=-\infty}^{\infty} \delta(f - i/d)$$

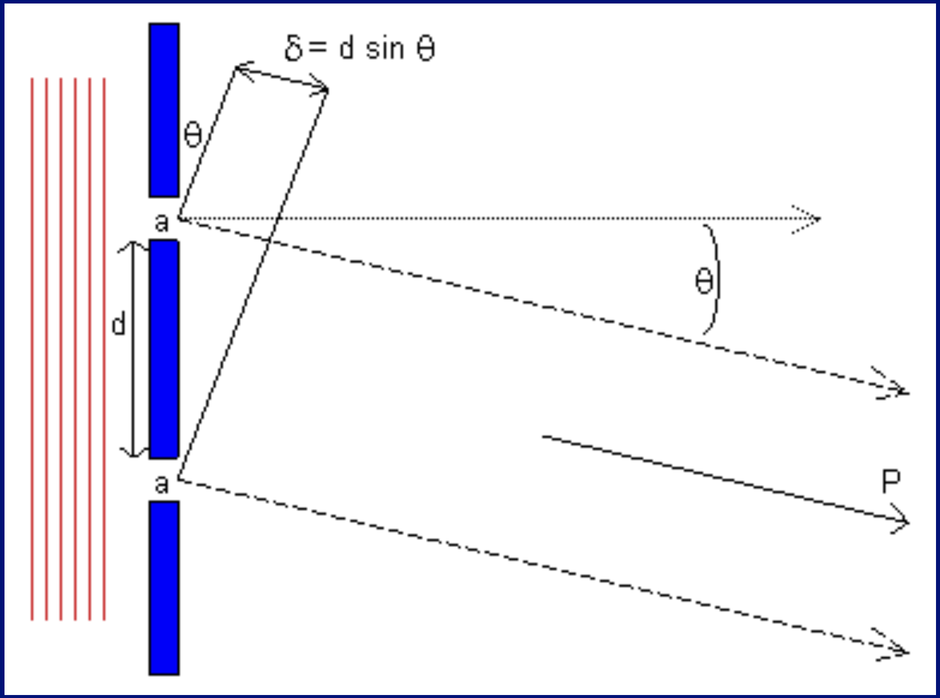


Radiation Pattern of a Sampled Aperture (II)

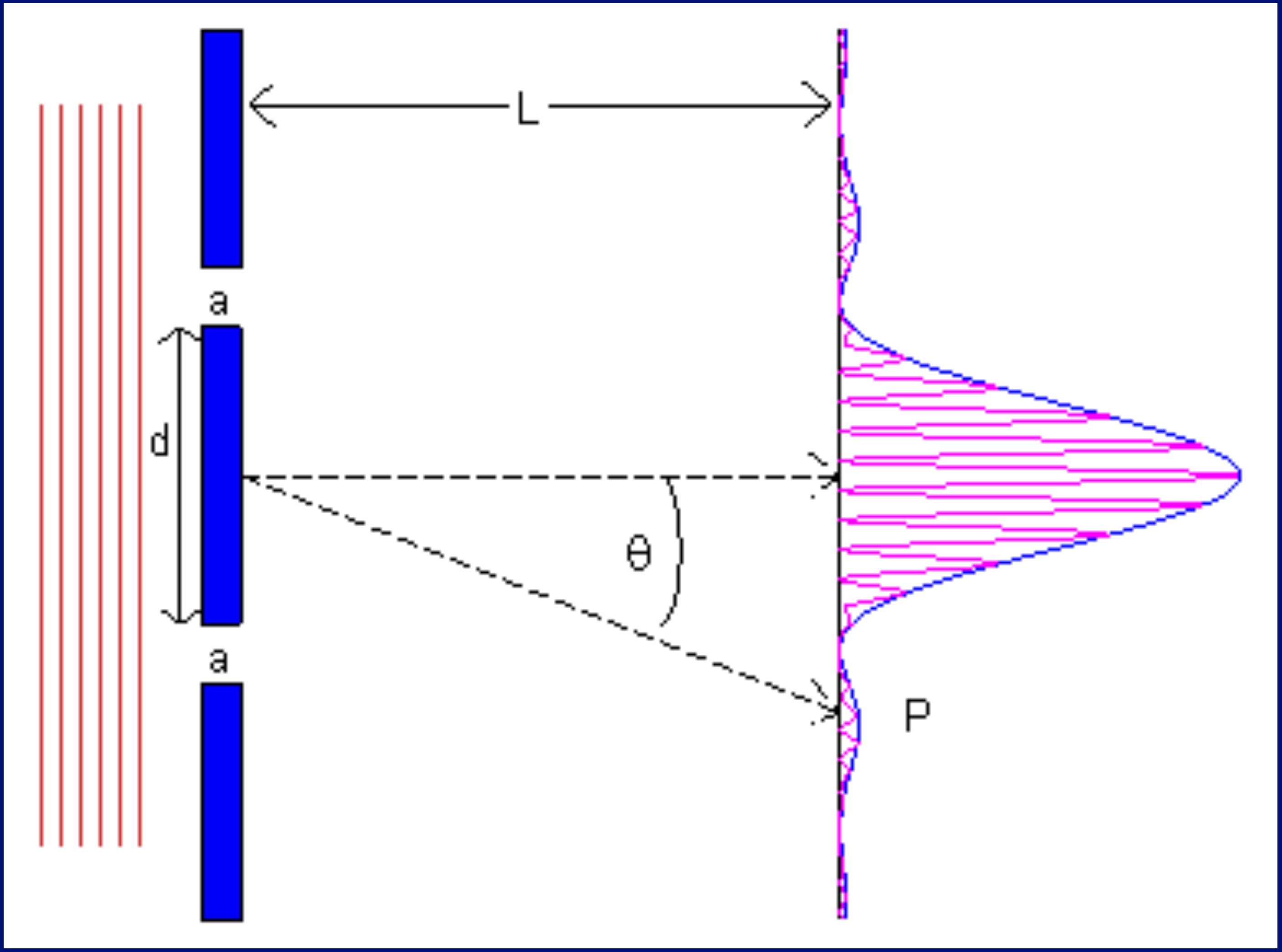


Interference

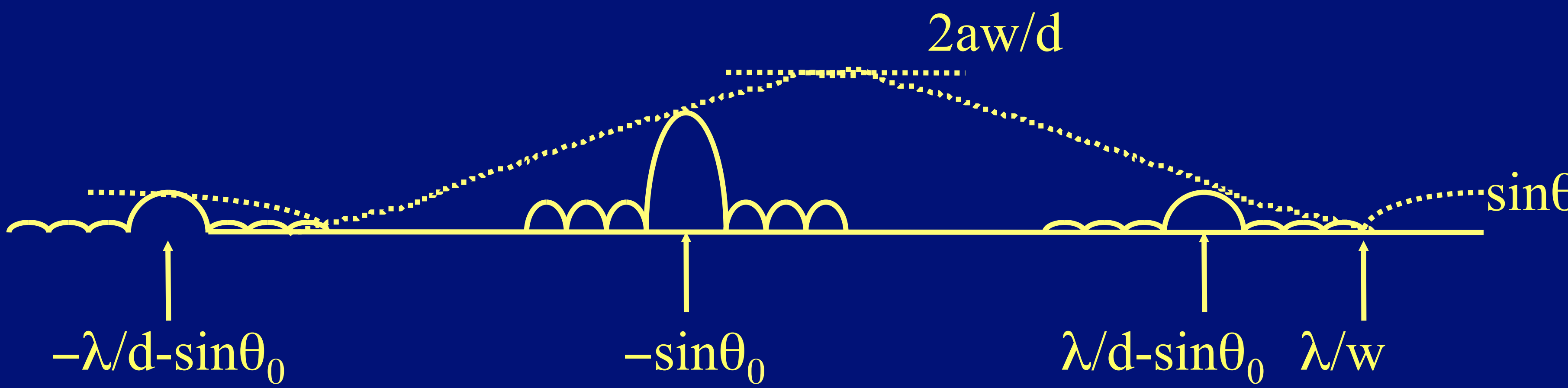


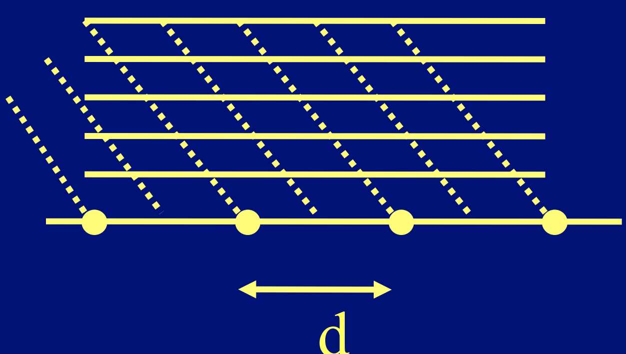


Interference



Array Steering and Grating Lobes





—— primary beam

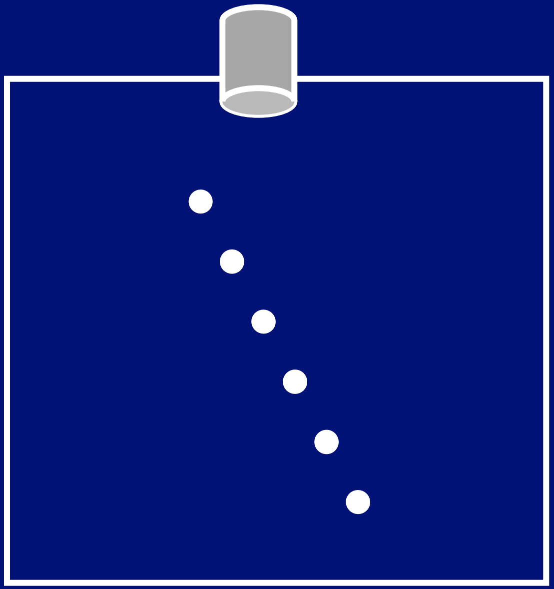
······ secondary beam

$$2 \le \lambda / d$$

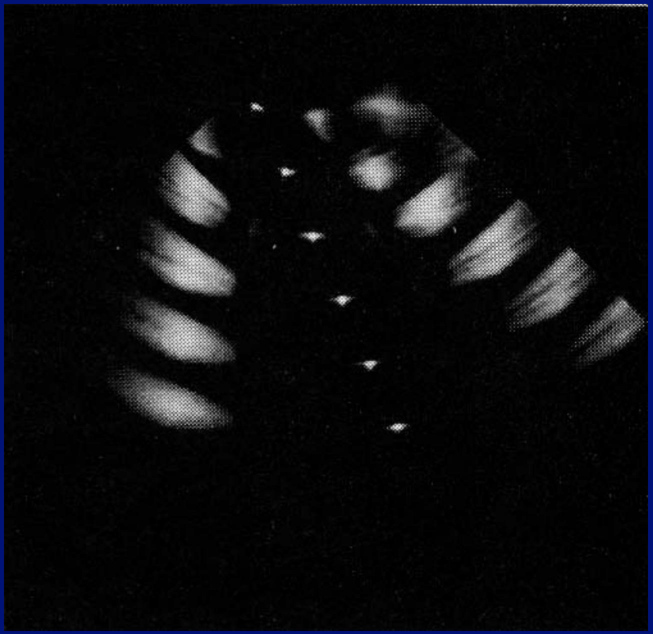
$$d \le \lambda / 2$$

Grating Lobes

No Grating Lobes



With Grating Lobes



Grating Lobes (II)

$$2\pi f_0 (\tau(x_i, R, \theta) - \tau(x_{i+1}, R, \theta)) > \pi$$

$$2\pi f_0 \left(\frac{R}{c} - \frac{x_i \sin\theta}{c} - \frac{R}{c} + \frac{x_{i+1}\sin\theta}{c}\right) = \frac{2\pi f_0 \sin\theta}{c}(x_{i+1} - x_i) = \frac{2\pi \sin\theta d}{\lambda} > \pi$$

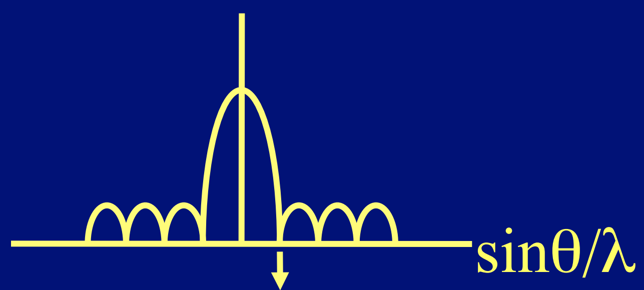
$$\sin\theta > \frac{\lambda}{2d} \quad \Rightarrow \quad d \leq \lambda / 2$$

Beam Sampling

spatial frequency
(aperture function)

radiation pattern

F.T.

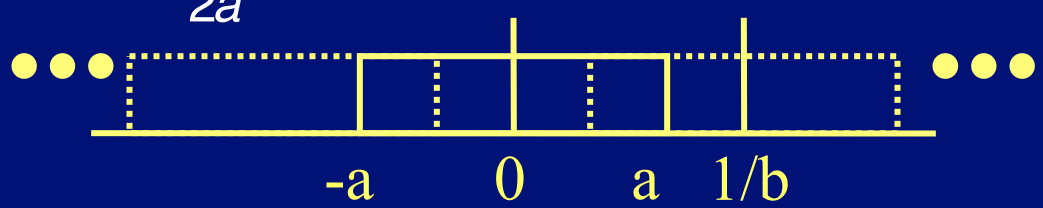


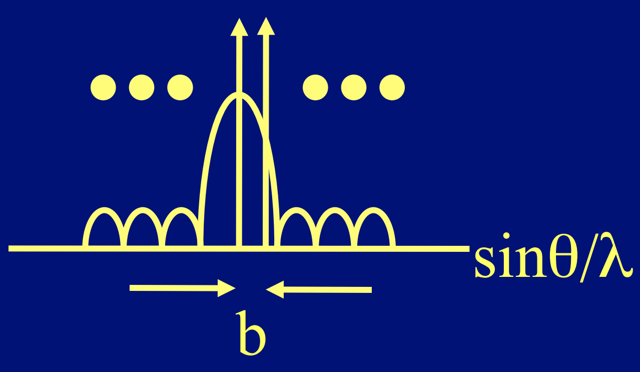
sampling

F.T.

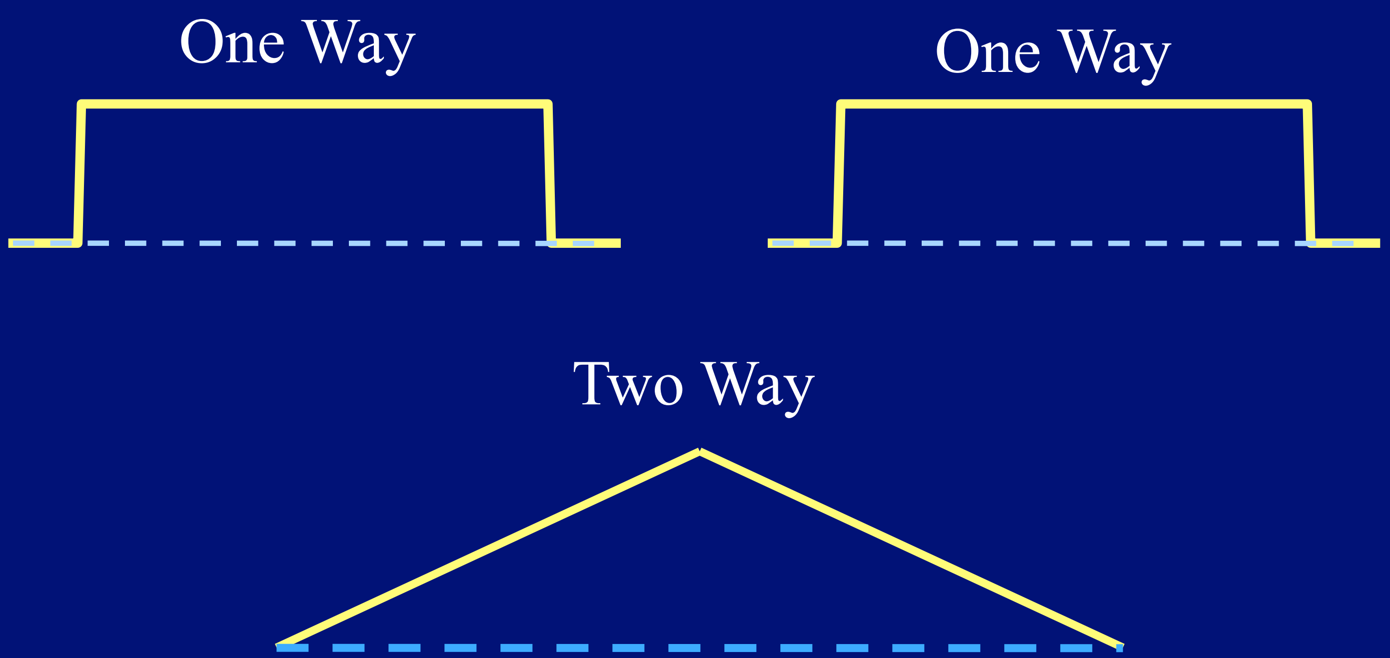
$$\frac{1}{\Delta \sin\theta / \lambda} \equiv \frac{1}{b} \geq 2a$$

$$\Delta \sin\theta \leq \frac{\lambda}{2a}$$





Beam Sampling



$$\Delta \sin\theta \le \frac{\lambda}{2a} \;\Rightarrow\; \Delta \sin\theta \le \frac{\lambda}{4a}$$

Beam Sampling (II)

One Way:

$$2\pi f_0\left(\tau(a,R,\theta_{i+1})-\tau(-a,R,\theta_{i+1})-(\tau(a,R,\theta_i)-\tau(-a,R,\theta_i))\right)$$

$$=\frac{2\pi f_0}{c}(2a\Delta\sin\theta)=\frac{2\pi f_0}{c}\frac{2a\lambda}{2a}=2\pi$$

$$\Rightarrow \quad \Delta\sin\theta\le\frac{\lambda}{2a}$$

2D Apertures

Diffraction for 2D Apertures

$$p(x',y',z)=\iint_{area}\frac{e^{jkd((x,y),(x',y'))}}{d((x,y),(x',y'))}dxdy$$

Diffraction for 2D Apertures

Separable aperture function:

$$C(x,y) = C(x)C(y)$$

$$B(x',y',z) = B(x',z)\times B(y',z) = F.T.\left[C(x)\right]\times F.T.\left[C(y)\right]$$

Circular aperture → Jinc radiation pattern.

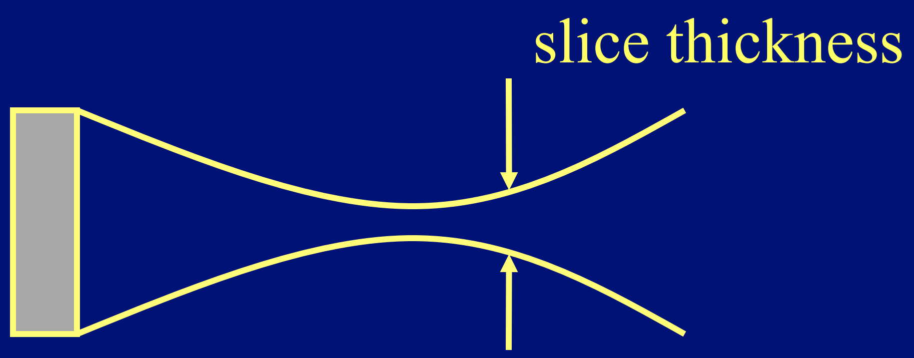
Two-Dimensional Transducer Arrays

Motivations: Elevational Focusing

- 1D arrays have only fixed elevational focusing capabilities. The elevational focusing quality is determined by a mechanical lens.
- 2D arrays must be used in order to have electronic, dynamic 3D focusing.

Motivations: Elevational Focusing

- Although not apparent, elevational focusing quality determines the "slice thickness" and is critical in image quality (including contrast resolution, noise characteristics and elevational resolution).



Motivations: Real-Time 3D Imaging

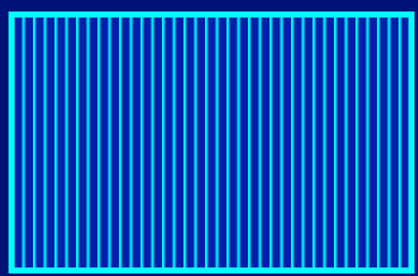
- Current 3D imaging (including flow imaging) is done by reconstruction using a set of 2D images. This is not real-time.
- Fully sampled (i.e., spatial Nyquist criterion is satisfied in both directions) arrays must be used to allow full 3D focusing and steering capabilities.

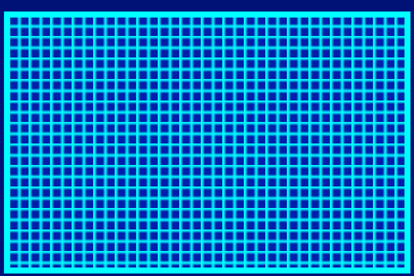
Complications

- For a fully sampled aperture with a typical size, the total channel count may exceed 10,000.
- As the element size decreases, electrical impedance significantly increases, thus resulting in poor signal to noise ratio.

Complications

- Inter-connection from each channel to the front-end electronics becomes complicated with 2D arrays.





Reduction of Channel Count

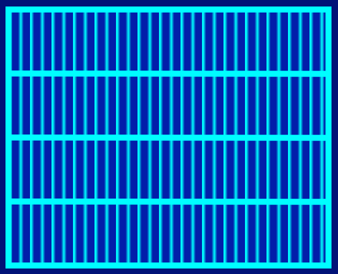
- Sparse array:
  - A random selection of elements are removed from the periodic dense array. Only a fraction of the original elements remains.
  - Mainlobe is un-affected. Grating lobes are avoided. Sidelobes are higher.
  - Electrical impedance is un-changed.
  - Difficult to manufacture.

Reduction of Channel Count

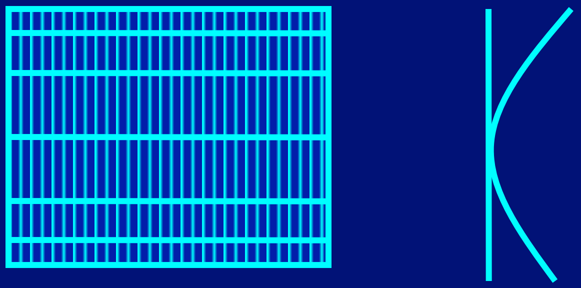
- 1.5D array:
  - Aperture in elevation is under-sampled.
  - Electrical impedance is reduced.
  - Total channel count is reduced.
  - Lack of elevational steering.
  - Full 3D focusing quality on the 2D image plane.
  - Also known as an-isotropic arrays.

Sampling of 1.5D Arrays

- Uniform sampling.



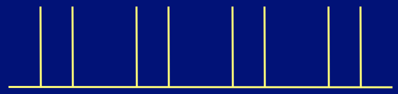
- Fresnel sampling.

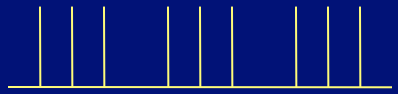


- Geometric delays are symmetric.

Sparse Periodic Arrays

- Despite of the periodicity, grating lobes are avoided by placing them at different locations.





Sparse Periodic Arrays

- Two-way beam pattern is determined by two-way aperture function, i.e., the convolution of the transmit aperture with the receive aperture.
- By carefully choosing the aperture functions, desired response can be synthesized.

Sparse Periodic Arrays

- Only valid for CW and in-focus. 4-to-1 reduction is reasonable.
- Signal-to-noise ratio is affected.
- In general, this is a synthetic aperture approach.
- This approach can be extended to 2D.

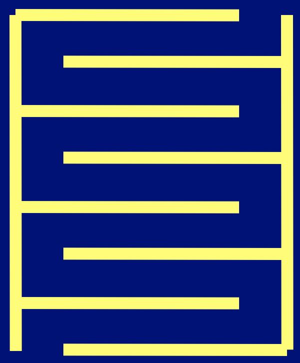
Lowering Impedance

- Electrical impedance significantly increases (high resistance and low capacitance) with small transducer area.
- For typical system characteristics, large impedance means poor sensitivity.
- Impedance can be reduced by many methods, including using multiple layer piezoelectric material.

Multi-Layer Ceramic

- Acoustically in series and electrically in parallel.

single layer



multiple layer

Multi-Layer Ceramic

- On transmit, the acoustic output pressure is increased by a factor of N (number of layers) assuming the same drive voltage.
- The output pressure is also increased by matching the impedance.

Multi-Layer Ceramic

- On receive, the received voltage is reduced by a factor of N assuming the same returning pressure.
- Capacitance is increased by a factor of N^2, due to the parallel connection and smaller distance between plates.
- The receive improvement depends on specific situations.

Multi-Layer Ceramic

- Both KLM and finite element models have been used to predict the performance and compared to measurement results.
- An alternative method is to have multiple layers on transmit and signal layer on receive.
- The coaxial cable capacitance can be reduced by integrating front-end with transducers.

Complications for Real-Time 3D Imaging

- Channel count.
- System complexity associated with increased channel count.
- The number of beams in a 3D imaging is dramatically increased, thus reducing "frame rate".
- Parallel beam formation is desired.

Homework #2

- Computer Homework #2: Beam Formation
- Due 12:00pm 4/24/2012 by emailing to

  paichi@ntu.edu.tw;

  r99945010@ntu.edu.tw

Homework #2

- Load hw2_dat.mat. In this data file, apertureU defines a uniform aperture and apertureH defines an aperture with non-uniform weighting. The spacing between two points in both cases is defined by d0 in mm. The vector pulseF defines a pulse spectrum of a particular excitation with the frequency axis specified by faxis (in MHz). Finally, the sound propagation velocity is defined by soundV in mm/usec. In all figures, please label all axes.

Homework #2

1. Assuming a continuous wave at 5MHz from the far field and zero incidence angle, plot the magnitude of the one-way diffraction patterns for both apertureU and apertureH (in dB). The horizontal axis should be $\sin\theta$ from –1 to 1. (20%)
2. Assuming a pulse wave which has the frequency response specified by pulseF and faxis, plot the magnitude of the one-way, far-field diffraction patterns (zero incidence angle) for both apertureU and apertureH (in dB). The horizontal axis should be $\sin\theta$ from –1 to 1. (20%)

Homework #2

3. Calculate the –6dB and –20dB mainlobe widths of the diffraction patterns obtained from 1 and 2. Comment on your answers and the sidelobe levels between two different apodization schemes. (20%)
4. Repeat 3 if the incidence angle is at 45 degrees. Justify your answers (20%)
5. Repeat 2 and 3 if the aperture is focused at 60 mm but the diffraction patterns are drawn at 55 mm and 65 mm, respectively. Comments on your answers. (20%)
6. (Bonus) Use the simulation programs to investigate dual beam formation (transmit/receive aperture, beam spacing).